DEFORMATION OF LEAD DISKS UNDER IMPACT

V. K. Bobolev, V. L. Bodneva,
and A. V. Dubovik

UDC 531.663

The effect of relatively low-velocity (1-3 m/sec) impact on a thin disk of imcompressible viscoplastic material placed in the gap between parallel rough surfaces is considered. The state of stress of the interlayer is assumed nearly hydrostatic during impact, the duration of which is limited by the elastic deformation of the elements of the striker system. The mathematical problem of determining the distributions of stresses, velocities, and temperatures for the axisymmetric deformation of a disk is reduced to the integration of an ordinary second-order differential equation. Numerical calculations for certain cases of impact are compared with the results of experiments on lead samples. Plane strain of an interlayer of viscoplastic material between rigid plates moving with a constant velocity is discussed in [1]. The state of stress of the interlayer for the same conditions of motion of the plates was studied in [2] for axial symmetry. In the present paper we take account of the impact nature of the loading and the elastic compression of the elements of the striker system, factors on which the deformation and the pressure developed in the impact depend.

We consider the axial compression of a thin disk of viscoplastic material placed between rough parallel plates. The thickness of the disk δ_0 is very much less than its radius R. Using the fact that δ/R is small, the equations of motion and continuity of the medium in cylindrical coordinates simplify to

$$\frac{\partial\sigma_r}{\partial r}+\frac{\partial\tau_{rz}}{\partial z}+\frac{\sigma_r-\sigma_\varphi}{r}=\rho\frac{du}{dt};\quad \frac{1}{r}\frac{\partial r\tau_{rz}}{\partial r}+\frac{\partial\sigma_z}{\partial z}=\rho\frac{dv}{dt};$$
$$\frac{du}{dt}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z};$$
$$\frac{dv}{dt}=\frac{\partial v}{\partial t}+u\frac{\partial v}{dr}+v\frac{\partial v}{dz};\quad \frac{1}{r}\frac{\partial ur}{\partial r}+\frac{\partial v}{\partial z}=0, \tag{1}$$

where u and v are the radial and axial components of the velocity, respectively; σ_i ($i = r, \varphi, z$) and τ_{rz} are the components of the stress tensor; and ρ is the density of the material.

We consider viscoplastic flow of the interlayer. Based on the picture of the state of stress found in [1] for a plane strain we assume that the tangential stress τ_{rz} reaches the yield point $\tau_s = \sigma_s/\sqrt{3}$ at the contact surfaces, where σ_s is the yield stress for uniaxial compression, assuming that the Mises yield criterion is satisfied. We assume that within the interlayer the tangential stresses are very much smaller than the normal stresses.

We use the fact that δ/R is small and average the equations of motion (1) over z from 0 to δ. Since $\tau_{rz} = \pm\tau_s$ at $z = 0$ and $z = \delta$, we obtain

$$\frac{\partial\bar{\sigma}_r}{\partial r}-\frac{2\tau_s}{\delta}+\frac{\bar{\sigma}_r-\bar{\sigma}_\varphi}{r}=\frac{d\bar{u}}{dt}. \tag{2}$$

From now on we omit the bars over quantities to denote averages.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 153-158, September-October, 1975. Original article submitted October 31, 1974.

Let $w = d\delta/dt < 0$ be the rate of axial displacement of the striker. Assuming that the contact surfaces remain plane during impact, we find that the equation of continuity is satisfied for the following values of the particle velocities averaged over the cross section of the layer:

$$u = -\frac{wr}{2\delta}, \quad v = \frac{wz}{\delta}. \tag{3}$$

It is known [3] that for axisymmetric flow of a viscoplastic body the components of the stress deviator are related to the components of the rate of strain tensor by the equation

$$\begin{pmatrix} \sigma_i' \\ \tau_{rz} \end{pmatrix} = \left(\frac{\tau_s}{I} + \mu\right)\begin{pmatrix} D_i \\ D_{rz} \end{pmatrix}; \tag{4}$$

$$D_r = D_\varphi = -\frac{1}{2} D_z = -\frac{w}{\delta}; \quad D_{rz} = 0; \quad I = -\frac{\sqrt{3}w}{\delta},$$

where μ is the plastic viscosity of the material which, as is the case with τ_s, is generally a function of temperature, pressure, and rate of strain [4]. We find from (4)

$$\sigma_r = \sigma_\varphi = \sigma_z + \sigma_s - \frac{3\mu w}{\delta}. \tag{5}$$

Taking account of (1), (3), and (5) and integrating (2) subject to the condition $\sigma_r(R) = 0$, we obtain

$$\sigma_r = \frac{2\tau_s}{\delta}(r - R) + \frac{\rho w^2 (r^2 - R^2)}{4\delta^2}\left(\frac{3}{2} - \frac{d \ln |w|}{d \ln \delta}\right). \tag{6}$$

The average pressure on the striker due to the interlayer is

$$p = -\frac{2}{R^2}\int_0^R \sigma_z r dr = \sigma_s + \frac{2\tau_s R}{3\delta} - \frac{3\mu w}{\delta} + \frac{\rho w^2 R^2}{8\delta^2}\left(\frac{3}{2} - \frac{d \ln |w|}{d \ln \delta}\right). \tag{7}$$

In (6) and (7) μ and σ_s must be considered as the effective values of the viscosity and yield stress of the material averaged over the radius of the disk. If these averages do not depend on pressure, temperature, and rate of strain, we can set $\mu = \mu_0$ and $\sigma = \sigma_s^0$. As $\mu \to 0$ and $\rho \to 0$ we find from (7) the value of the average stress for a rigid-plastic body [4]. Assuming that the contract surfaces are smooth and that the second term in (7) is absent, we arrive at the expression obtained in [2].

We consider impact on the viscoplastic interlayer with relatively low velocities $|w_0|$. We have from (7)

$$p = \sigma_s + \frac{2\tau_s R}{3\delta} - \frac{3\mu w}{\delta}. \tag{8}$$

Assuming that the displacement of the center of gravity of the striker is made up of the decrease in the thickness of the interlayer and the elastic compression of the striker, from Newton's second law we find

$$\frac{d^2}{dt^2}\left(\delta - \delta_0 - \frac{pS}{k}\right) = \frac{pS}{M}, \quad S = \pi R^2. \tag{9}$$

Here k is the ultimate rigidity of the elements of the striker system and depends somewhat on the energy of the impact, and M is the mass of the striker. In writing (9) we assume that at t = 0 the compression of the striker is negligible in comparison with the thickness of the disk, i.e.,

$$d = \frac{\sigma_s S}{k\delta_0}\left(1 + \frac{2R}{3\sqrt{3}\delta_0} - \frac{3\mu w_0}{\sigma_s \delta_0}\right) \ll 1.$$

We introduce the dimensionless variables $\tau = -w_0 t/\delta_0$, $x = \delta_0/\delta$, $y = w/w_0$, $P = p/\sigma_s$ and the quantities $\alpha = \sigma_s S/k\delta_0$, $\beta = \sigma_s S\delta_0/Mw_0^2$, $\gamma = 2R/3\sqrt{3}\delta_0$, $\varepsilon = -3\mu w_0/\sigma_s\delta_0$. Then the flow parameters in the impact of the interlayer can be found by solving the system

$$\frac{d^2}{d\tau^2}(x^{-1} - 1 - \alpha P) = \beta P, \quad P = 1 + \gamma x + \varepsilon xy, \tag{10}$$
$$\frac{dx}{d\tau} = x^2 y, \quad x(0) = 1, \quad y(0) = 1, \quad y(0) = -\beta(1 + \gamma + \varepsilon).$$

System (10) leads to the second-order differential equation

$$\alpha\varepsilon \frac{d}{dx}\left(yx^5 \frac{dy}{dx}\right) + yx^4(\alpha\gamma x^2 + 1)\frac{dy}{dx} + 2\alpha\varepsilon x^3 y^3 + 2\alpha\gamma x^3 y^2 + \beta\varepsilon yx + \beta\gamma x + \beta = 0, \tag{11}$$
$$y(1) = 1, \; y'(1) = -\beta(1 + \gamma + \varepsilon).$$

For $\varepsilon = 0$ we find from (11) the law of solving down of the striker by the layer of rigid-plastic material [5]:

$$y = \frac{[(1 - \alpha\gamma)^2 - \alpha\beta\gamma^2(x^2 - 1) - 2\beta\gamma \ln x - 2\alpha\beta\gamma(x - 1) - 2\beta(1 - x^{-1})]^{1/2}}{1 + \alpha\gamma x^2}. \tag{12}$$

For an absolutely rigid striker ($\alpha = 0$) from (12) we obtain

$$y = [1 - 2\beta\gamma \ln x - 2\beta(1 - x^{-1})]^{1/2}.$$

We turn to a discussion of the heating of the spreading layer. For axisymmetric motion of a viscoplastic medium the heat flux equation for small δ/R has the form

$$\rho c_p \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + v\frac{\partial T}{\partial z}\right) = \lambda\frac{\partial^2 T}{\partial z^2} + \mu I^2 + \tau_s I, \tag{13}$$

where λ and c_p are the thermal conductivity and specific heat of the material. If $\rho c_p |w|\delta/2\lambda \gg 1$, the first term on the right-hand side of (13) can be neglected practically up to the instant the striker comes to rest. We pose the problem of calculating the maximum temperature averaged over the radius and thickness of the interlayer. Integrating (13) under the condition $T(\delta_0) = 0$ and using (4), we obtain

$$T = T_1 + T_2 = \frac{\sigma_s}{\rho c_p}\ln x - \frac{3\mu w_0}{\rho c_p \delta_0}\int_1^x y(\xi)\,d\xi. \tag{14}$$

Here the first term T_1 characterizes the plastic heating of the medium and the second T_2, the contribution of viscous heating.

Equations (10) and (14) were solved numerically by computer using the standard fourth-order Runge–Kutta program. The accuracy of the calculation was 0.5-1%.

We turn to an analysis of the data on impact experiments with thin disks (δ_0 = 1 mm, R = 5 mm) of technical lead. The experimental procedure is similar to that described earlier [6] in connection with investigations of the laws of the impact fracture of thin samples of brittle low-strength materials. The

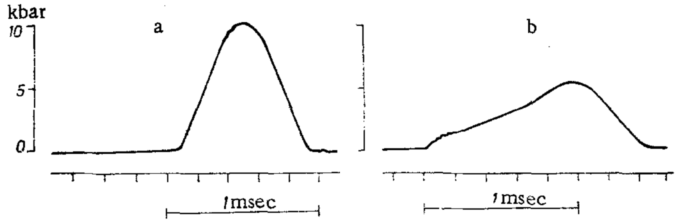

Fig. 1

TABLE 1

$\vert w_0\vert$, m/sec k, kbar · cm	1 130	1.5 150	2 150	3 160
p_x, kbar	4.7	7.5	10,1	15,5
t_x, msec	0,90	0.83	0,83	0,80
p_m, kbar	1,6	2,8	5,3	12,3
t_m, msec	0,96	1,07	0,94	0,75
ln x_k	0.71	1,45	2,29	2,79

TABLE 2

m/sec	1	1,5	2	3
p_m, kbar	2,05	3,29	5,67	11,93
t_m, msec	0,723	0,821	0,785	0,662
ln x_k	0,478	1,065	1,681	2,475

necessary information on the character of the loading of the lead disks was derived from an analysis of pressure–time oscillograms obtained by using a wire strain gauge and by measuring the maximum compression of the interlayer x_k after the impact. The samples under study were placed between the flat ends of 10-mm-diameter roller bearings. The experiments were performed on a vertical drop hammer with a freely falling 10-kg load moving with a velocity $|w_0|$ = 1-3 m/sec which applied an axial blow through the rollers.

Figure 1a and b shows oscillograms of the pressure during impact with no material between the rollers ("empty collision") and in an experiment with a lead disk ($|w_0|$ = 2m/sec; the time marks are 200 μsec apart). The presence of the plastic interlayer increases the time of impact by more than 50% and nearly halves the maximum pressure. Three characteristic parts can be distinguished on the pressure–time curve shown in Fig. 1b. During a short initial period $t_1 \sim 100$ μsec the pressure rises rapidly to $\sim$ 1 kbar. The rate of increase of pressure then decreases somewhat and from the time $t_2 \sim 600$ μsec it again reaches approximately its initial value. This behavior of the curve can be related to the fact that at the start of the impact the deformation of the sample is nearly elastic. When the lead in an inner annular zone reaches the yield point the sample is transformed to the plastic state [4]. With a further increase in pressure the plastic zone expands, and from time t_1 developed plastic (viscoplastic) flow begins in practically the whole interlayer. The average pressure corresponding to this instant is given by Eq. (8). As the thickness of the disk is decreased the resistance to the motion of the striker increases rapidly and from time t_2 the energy of the weight is predominantly expended in the compression of the striker system. The further behavior of the curve in Fig. 1b is very similar to the "empty collision" oscillogram (Fig. 1a) with the time of fall off of the pressure from the maximum p_m approximately the same in the two cases.

Table 1 presents the results of measurements of certain collision parameters in experiments with lead samples. Here p_x and t_x are the maximum pressure and the time of an empty collision, and t_m is the time to reach the maximum pressure p_m in the experiments with lead. Calculations performed with Eq. (10) for $\varepsilon = 0$ (model of a rigid-plastic body) show (Table 2) that the measured values of p_m agree satisfactorily with the theoretical if we set σ_s = 0.5 kbar. However, the calculated values of x_k differ appreciably from the experimental.

TABLE 3

$\vert w_0\vert$ m/sec σ_s, kbar μ, П	1 0,3 $0,7 \cdot 10^5$	1.5 0,3 $0,7 \cdot 10^5$	2 0,27 10^5	3 0.4 10^4
p_m, kbar	1,51	2,90	5,26	12,20
t_m, msec	0,865	0,917	0,834	0.683
ln x_k	0,708	1,487	2,240	2,738
T_1, K	15,0	31,5	42,7	77,0
T_2, K	23,8	68,6	178	105

There is good agreement between measured collision parameters and those calculated with the model of a viscoplastic medium (Table 3) if different σ_s and μ are used for different values of $|w_0|$, keeping them within reasonable limits close to the experimental values [2, 7].

Figure 2 shows curves calculated from (10) and (14) for p (1), x (2), and y (3) as functions of τ for $|w_0|$ = 2 m/sec ($\alpha = 0.0131$, $\beta = 0.0491$, $\gamma = 1.9245$, $\varepsilon = 2.4$, so that $d = 0.0697$). The curve for $p(\tau)$ closely coincides with the experimental (open) curve if for the time of the initial stage of the impact, which is not taken into account in the theory, is assumed $\tau_1 = 0.2$ or $t_1 = 0.1$ msec. The dashed-dot part of curve 1 is

TABLE 4

$\lvert w_0 \rvert$, m/sec	1.5	2	3
kbar · cm	1500	1500	1600
p_m, kbar	3,06	7,02	30,96
t_m, msec	0,805	0,696	0,474
$\ln x_k$	1,535	2,518	3,682

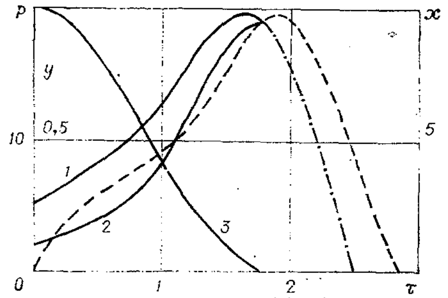

Fig. 2

plotted from experimental data under the assumption of elastic relief of the elements of the striker system.

A certain regularity is observed in the sequence of values of σ_s and μ used in the calculations. These are the values of the yield point and the viscosity of the material of the interlayer averaged over the time of the impact. For small impact velocities of 1-2 m/sec σ_s and μ are approximately constant and vary only for $|w_0| = 3$ m/sec when the range of pressures and the role of dissipative effects are noticeably increased. Thus, if $\mu = 10^5$ p is used to calculate T for the last case in Table 3 we obtain a temperature of 329°C, which is the melting point of lead under normal pressure, instead of 105°C. Therefore, the increase of σ_s with increasing $|w_0|$ can be related mainly to the strengthening of lead, and the decrease in μ to the heating of the interlayer. Table 4 shows how important it is to take account of the deformation of the elements of the striker system. The table gives the collision parameters calculated with the values of σ_s and μ given in Table 3, but with the rigidity k of the striker equal to 10 kbar · cm.

LITERATURE CITED

1. V. P. Myasnikov, "On the compression of a viscoplastic layer by rigid plates," Izv. Akad. Nauk SSSR, Mekh. Mashinostr., No. 4 (1963).
2. J. B. Haddow, "The compression of a thin disk," Int. J. Mech. Sci., 7, No. 10 (1965).
3. P. M. Ogibalov, and A. Kh. Mirzadzhanzade, Nonstationary Motions of Viscoplastic Media [in Russian], Izd. MGU (1970).
4. L. M. Kachanov, Fundamentals of the Theory of Plasticity [in Russian], Nauka, Moscow (1969).
5. A. V. Dubovik and V. K. Bobolev, "Ultimate compression of liquid and plastic interlayers during the collision of elastic bodies," Dokl. Akad. Nauk SSSR, 210, No. 6, 1331 (1973).
6. G. T. Afanas'ev, V. K. Bobolev, and A. V. Dubovik, "Deformation and fracture of a thin disk under compression," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1971).
7. Handbook for Metal Workers. T.Z.M. [in Russian], Mashinostroenie, Moscow (1959).